Announcement: HW3 out today

* Concentration
* "Modern concern" - Higher order moments
* Hoeffding / Sub-Gaussian
* Rademacher, why?
* Rademacher for DL
* Rademacher Basics
* Interpolation/Double descent

Theorem (Hoeffding). Suppose \((Z_1, \ldots, Z_n)\) are independent, \(Z_i \in [a_i, b_i]\),
\[
\Pr \left[ \left| \frac{1}{n} \sum Z_i - \mu \right| > \epsilon \right] \leq 2 \exp \left( -\frac{2 \epsilon^2 n}{4 \mu^2} \right).
\]

Proof. \(\sum Z_i \) is a \(\mathbb{Z}_i\)-sub-Gaussian function of \(X \in \mathbb{R}^n\).

\begin{align*}
\Pr \left[ \frac{1}{n} \sum Z_i > \epsilon \right] &
\leq \Pr \left[ \exp(t \frac{1}{n} \sum Z_i) > \exp(\frac{\epsilon}{n}) \right] \\
&\leq \exp \left( -\frac{\epsilon^2}{2 \sigma^2} \right) \leq \exp \left( -\frac{\epsilon^2}{2 \mu^2} \right)
\end{align*}

\[
\Pr \left[ \frac{1}{n} \sum Z_i < -\epsilon \right] \leq \Pr \left[ \exp(t \frac{1}{n} \sum Z_i) < \exp(-\frac{\epsilon}{n}) \right] \\
&\leq \exp \left( -\frac{\epsilon^2}{2 \sigma^2} \right) \leq \exp \left( -\frac{\epsilon^2}{2 \mu^2} \right)
\]

Examples: Last term, but \((X_i, -X_i)\) is \(a_i - b_i\)-sub-Gaussian.

\[
\frac{1}{n} \sum Z_i \] is \(a_i - b_i\)-sub-Gaussian.

Character bounding technique.

Lemma (Markov). \(X \geq 0\) a.s., \(a > 0\).
\[
\Pr \left[ X \geq a \right] = \frac{a}{b}.
\]

Proof: Note \(\Pr[ X \leq a ] \leq X \), apply Chernoff.

Charater bounding technique:

\[
\Pr \left[ X \geq \epsilon \right] = \Pr \left[ \exp(tX) \geq \exp(t\epsilon) \right] \\
= \Pr \left[ \exp(tX) \geq \exp(t\epsilon) \right] \\
\leq \exp \left( -\frac{\epsilon^2}{2 \mu^2} \right)
\]

One more tail inequality

Theorem (McDiarmid). Suppose \((Z_1, \ldots, Z_n)\) are independent
\[
\sup_{-\epsilon_i \leq Z_i \leq \epsilon_i} \left| F(Z_1, \ldots, Z_n) - F(Z_1, \ldots, Z_n) \right| \\
\leq \epsilon_i
\]

Then \(F(Z_1, \ldots, Z_n)\) is \(\mathbb{Z}_i\)-sub-Gaussian.
Redes de orden superior - complejos

**Theorem (Ladder ladder kernel)**

Let $G$ be a graph with $G(z)$ and $G(L)$ m-regular. Then

$$\text{Spectrum}(G(z) + 2I) = \frac{2}{n} \text{Spectrum}(G(L)) + (i\alpha)^{\frac{2n}{n}}$$

**Remarks**

1. **[Too many]** too many classical runs by [Bakker-Heidenhain Conjecture].
2. Many analogous properties, but most graph polynomials can be obtained as
4. Actual problem solved in a universal approach in [Rothschild-Zaslavsky].

**Proof (Sketch)**

Let $G(L)$ be given with $G(z) = z^L$. Then

$$\text{Spectrum}(G(z) + 2I) = \frac{2}{n} \text{Spectrum}(G(L)) + (i\alpha)^{\frac{2n}{n}}$$

**Lemma**

1. $\text{Spectrum}(G(L)) = \chi_G$
2. $\text{Spectrum}(G(z)) = \text{Spectrum}(G(L))$.
3. Let $\text{Spectrum}(G(L))$ be given with $G(z) = z^L$. Then

**Problem**

Show for any $G$, $z^L$, and $\chi_G$

$$\text{Spectrum}(G(L)) = \chi_G$$